VLBI Measurements of the Crustal Deformations Induced by Polar Motion

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Abstract—The Earth's crustal deformations induced by polar motion were revealed by processing the 4-year-long observations of extragalactic sources using very long baseline interferometry (VLBI) networks within the framework of the geodetic IRIS program. The estimates of the Love numbers are found for polar motion: $h_p = 0.65 \pm 0.20$ and $\ell_p = 0.11 \pm 0.05$. These estimates fit the theoretical values well.

INTRODUCTION

The Earth is an ellipsoid of revolution, rather than a sphere, due to its response to the centrifugal force caused by diurnal rotation. Variations in the rotation rate and position of the rotation axis change this force and thereby deform the surface of the Earth. Since the relative variations in the Earth's rotation rate do not exceed 10^{-8}, and the variations in the polar coordinates are about $1–4 \times 10^{-6}$ radians (0″2–0″8), the change of the centrifugal force is mainly related to the change in the rotation axis position within in the body of the Earth. The variation in the centrifugal potential due to the displacement of the rotation pole induces the deformations called polar motion [1]; similarly, the variable tidal potential induces the variable tidal deformations of the crust.

Wahr [2] noticed that these deformations can reach a few centimeters and can, in principle, be measured. Although the changes in the observing station coordinates, which are caused by polar motion, have not yet been found by direct measurements, the International Earth Rotation Service (IERS) proposed, in 1992 [3], to introduce a polar motion correction when processing high-precision astronomical observations.

Indirect evidence of polar motion is an increase in the Chandler period compared to the period of free polar motion in the model of absolutely rigid body. This increase is mainly caused by the variation in the inertia tensor due to rotational deformations in the figure of the Earth. It is determined by the global redistribution of masses and is proportional to Love number k [1]. This present paper attempts to determine the local deformations induced by polar motion from direct VLBI observations. These deformations are proportional to Love numbers h and ℓ, as will be shown below.

THEORY

According to the classical tide theory of an elastic and oceanless Earth, the displacement vector due to disturbing potential V can be written in the following form [4]:

$$u_v = \sum_{n=2}^{n=\infty} \frac{h_n}{g} V_n,$$

$$u_e = \sum_{n=2}^{n=\infty} \frac{\ell_n}{g \cos \varphi} \frac{\partial V_n}{\partial \lambda}, \qquad (1)$$

$$u_n = \sum_{n=2}^{n=\infty} \frac{\ell_n}{g} \frac{\partial V_n}{\partial \varphi},$$

where u_v, u_e, and u_n are, respectively, vertical, easterly, and northerly displacement components in the topocentric coordinate system; g is local gravity; φ is the geocentric latitude; λ is the longitude; V_n is the nth harmonic in the expansion of the disturbing potential in spherical functions; h_n and ℓ_n are the dimensionless Love numbers of the nth order, characterizing response of the Earth to the nth harmonic of the disturbing potential.

The centrifugal potential is

$$V_c = \frac{1}{2} (\mathbf{R} \times \mathbf{\Omega})^2, \qquad (2)$$

where $\mathbf{R}$ is the geocentric vector of the station coordinates and $\mathbf{\Omega}$ is the vector of angular velocity of the Earth. Expressing the angular velocity in polar coordinates $\mathbf{\Omega} = \omega_0 (X_p, -Y_p, 1)^\top$ and neglecting the squares of
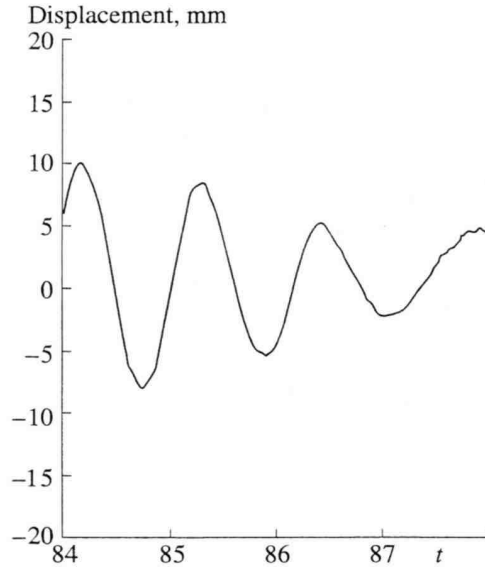
Displacement, mm

Fig. 1. The calculated vertical displacement of the coordinates of the Wettzell station due to the polar motion.
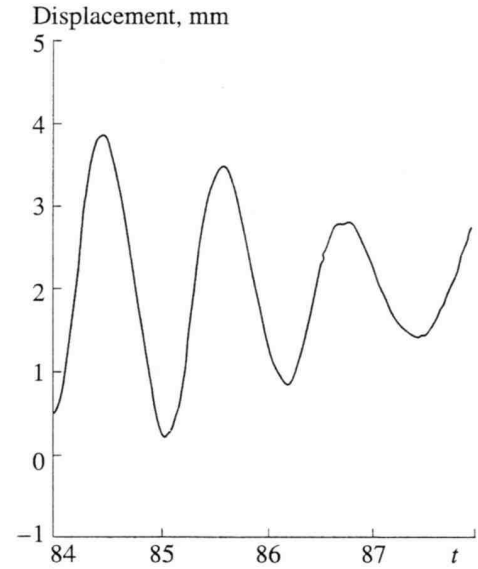
Displacement, mm

Fig. 2. The calculated eastward displacement of the coordinates of the Wettzell station due to the polar motion.

coordinates of the pole, we can transform (2) into the form

$$V_c(\mathbf{R}, \mathbf{\Omega}) = \frac{1}{2}\omega_0^2 R^2$$
$$\times [(r_x^2 + r_y^2) - 2r_z(r_x X_p - r_y X_p)], \quad (3)$$

where $\mathbf{r} = \dfrac{\mathbf{R}}{|R|}$. The first term in (3) does not depend on time, and the corresponding deformation is included in the station coordinates; therefore, this term is not considered below. The variable component of the centrifugal potential in spherical coordinates has the form

$$V_{pt}(R, \varphi, \lambda, \omega_0) = -\omega_0^2 R^2 \sin\varphi\cos\varphi$$
$$\times (X_p\cos\lambda - Y_p\sin\lambda). \quad (4)$$

Since only the terms of the second order of smallness are retained in the expansion of the centrifugal force in spherical functions, the final expression for the displacement induced by the polar motion has the form

$$u_v = h_2\frac{\omega_0^2 R^2}{g}\sin\varphi\cos\varphi(Y_p\sin\lambda - X_p\cos\lambda),$$

$$u_e = \ell_2\frac{\omega_0^2 R^2}{g}\sin\varphi(Y_p\cos\lambda + X_p\sin\lambda), \quad (5)$$

$$u_n = \ell_2\frac{\omega_0^2 R^2}{g}\sin 2\varphi(Y_p\sin\lambda - X_p\cos\lambda),$$

where h_2 and ℓ_2 are the Love numbers of the second order. Their nominal values, calculated by Wahr [5] for

the Earth model 1066A, are 0.609 and 0.0852, respectively. In a more complicated Earth model incorporating the mantle anelasticity and liquid core, the Love numbers are frequency-dependent. In particular, the Love numbers increase with a decrease in the frequency of the disturbing potential and, moreover, this growth is sensitive to the unreliably determined parameters of the internal structure of the Earth. The annual and Chandler ($P \sim 430$ days) components are known to dominate the spectrum of polar motion [1]. The expected increase of Love number h_2 is 4–6% for periods of about one year. For this reason, the experimental determination of the Love numbers for low frequencies is of great interest.

We emphasize that the effect of polar motion is small. Figures 1 and 2 show the coordinate displacements for the Wettzell station, calculated for the period 1984–1987.

OBSERVATIONS AND RESULTS

In order to determine the Love numbers for polar motion, the 4-year-long (1984–1987) series of geodetic VLBI observations was processed. The data obtained within the framework of the IRIS program [6] were kindly afforded by K. Noll from the Goddard Center of Space Flights, NASA. The observation series contains 284 diurnal intervals of observations at three to five stations equipped with the hydrogen frequency standard and the Mark-III data acquisition system (see table). The 24-hour, X- and S-band observations of 19 extragalactic compact radio wave sources, consisting of 200 to 900 scans, were carried out every five days. The difference between the travel times of a signal to the network

stations was measured [7]. In all, 117 361 measurements of the group delay were involved in the analysis.

The observations were processed by the VORIN software developed at the Institute of Applied Astronomy [8, 9]. Reducing the observations, we followed the IERS 92 standard with the following exceptions: (1) polar motion correction was not considered; (2) the exact expressions of Wahr, allowing for the frequency dependence of the Love number, were used to calculate the tide of the second-order for 22 tidal waves; (3) the correction for a lunar tide of the third order was introduced; (4) subdiurnal variations of UT1 and pole coordinates were treated in terms of the model of Herring and Dong [10]. The ionospheric correction was calculated on the basis of the delay measurements at X- and S-bands. The tropospheric delay at the zenith was calculated by the Saastamoinen formula [11] from the surface values of meteoparameters. The height dependence of the tropospheric delay was modeled by the map function of Niell [12].

The multi-group weighted least-squares method was used to estimate the model parameters. The Love numbers for polar motion, the coordinates of sources, the velocities of stations, and their initial coordinates were calculated. These parameters were assumed to be constant during the entire period of observations. Moreover, the corrections to the following Earth rotation parameters were found for each series: UT1, rotation velocity of the Earth, pole coordinates of nutation angles, tropospheric delay at the zenith (for each station), and station clock characteristics (except the clocks of the reference station). The tropospheric delay at the zenith and clock function were modeled by a linear spline with a segment duration of about 2 h. The estimation included stabilizing constraints imposed on the equality between the variation rates of the tropospheric delay at the zenith for adjacent segments with a dispersion of $(50 \text{ ps/h})^2$ and equality between the rates of the clock drift with a dispersion of $(50 \text{ fs/h})^2$.

The estimates of the Love numbers obtained for polar motion are

$$h_p = 0.65 \pm 0.20,$$

$$\ell_p = 0.11 \pm 0.05.$$

They are rather close to the theoretical values given above. The weighted rms deviation of the residuals is 35.2 ps. If the Love numbers for polar motion are ignored and set to zero, the rms deviation is 35.7 ps.

CONCLUSIONS

1. The direct experimental confirmation of the displacement of the coordinates of observation stations due to polar motion is derived.

2. The values of Love numbers h and ℓ obtained for polar motion agree well with the model of an elastic oceanless Earth. The accuracy of determination of the Love numbers from the 4-year-long observation series is still inadequate to estimate the effect of the Earth's anelasticity on their values. Additional observations are required to improve the accuracy by a factor of three to five.

REFERENCES

1. Morits, M.G. and Myuller, A., *Vrashchenie Zemli: teoriya i nablyudeniya* (Rotation of the Earth: Theory and Observations), Kiev: Naukova Dumka, 1992.

2. Wahr, J.M., Deformation Induced by Polar Motion, *J. Geophys. Res.*, 1985, vol. 90, no. B11, p. 9369.

3. IERS Standards, MacCarthy, D.D., Ed., *IERS Technical Note*, 1992, no. 13.

4. Melchior, P., *The Tides of the Planet Earth*, Oxford: Pergamon, 1983, p. 662.

5. Wahr, J.M., Body Tides on an Elliptical, Elastic, Rotating and Oceanless Earth, *Geophys. J. Int.*, 1981, vol. 64, no. 3, p. 677.

6. Carter, W.E. and Robertson, D.S., Project POLARIS and IRIS: Monitoring Polar Motions and UT1 by Very Long Baseline Interferometry, *Space Geodesy and Geodynamics*, Anderson, A.J. and Cazenave, A., Eds., Academic Press, 1986, p. 221.

7. Thompson, A., Moran, J., and Swenson, G., Jr., *Interferometry and Synthesis in Radio Astronomy*, New York: Wiley, 1986. Translated under the title *Interferometriya i sintez v astronomii*, Moscow: Mir, 1989.

8. Petrov, L.Yu., *Vtorichnaya obrabotka geodezicheskikh RSDB nablyudenii. Model' reduktsii* (Secondary Processing of Geodetic VLBI Observations: A Reduction Model), St. Petersburg: IPA, *Soobshcheniya IPA* (IAA Reports), no. 74–75, 1995, p. 62.

9. Petrov, L.Yu., *Vtorichnaya obrabotka geodezicheskikh RSDB nablyudenii. Otsenivanie parametrov modeli* (Secondary Processing of Geodetic VLBI Observations: Estimation of Model Parameters), St. Petersburg: IPA, *Soobshcheniya IPA* (IAA Reports), no. 76, 1995, p. 47.

10. Herring, T.A. and Dong, D., Measurement of Diurnal and Semidiurnal Rotational Variations and Tidal Parameters of the Earth, *J. Geophys. Res.*, 1994, vol. 99, no. B9, p. 18051.

11. Saastamoinen, J., Atmospheric Correction for the Troposphere and Stratosphere in Radio Ranging of Satellites, *The Use of Artifical Satellites for Geodesy, Geophys. Monography Ser. 15, AGU*, Washington: AGU, 1972, p. 247.

12. Niell, A.E., A New Approach for the Hydrostatic Mapping Function, in *Proceedings of the International Workshop for Reference Frames. Establishment and Technical Development in Space Geodesy*, Tokyo, 1993, p. 61.